Extended Abstract: The Port-Hamiltonian Structure of Vehicle-Manipulator Systems

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ABSTRACT

Vehicle-manipulator systems (VMS)—including aerial manipulators, underwater manipulators, space robots, and mobile manipulators—are typically modeled using the Euler–Lagrange formalism, which obscures the underlying energetic structure. This paper presents a port-Hamiltonian formulation of VMS dynamics derived from first principles using Hamiltonian reduction theory. Exploiting the principal bundle structure of the VMS configuration space, we derive an inertially-decoupled port-Hamiltonian model that explicitly reveals the energy flow, passivity, and geometric structure of these systems, opening new avenues for energy-based control, structure-preserving simulation, and physics-informed learning.

1. Introduction and Motivation

Vehicle-manipulator systems (VMS) are robotic systems comprising a manipulator mounted on a mobile base, including aerial manipulators, underwater manipulators, space robots, and mobile manipulators.

Lagrangian formulations and their limitations. VMS dynamics are conventionally derived through Newton–Euler recursion [1] or variational principles, yielding Euler–Lagrange equations

$$\mathcal{M}(q)\begin{pmatrix}\dot{\mathrm{v}}\\ \ddot{q}\end{pmatrix} + \mathcal{C}(\mathrm{v},q,\dot{q})\begin{pmatrix}\mathrm{v}\\ \dot{q}\end{pmatrix} + g(q) = \begin{pmatrix}\mathrm{w}\\ \tau\end{pmatrix}, \qquad (1)$$

where $q \in Q_m$ denotes the joint configuration, v and $\dot{q}$ are the base and joint velocities, $\mathcal{M}(q)$ is the inertia matrix, $\mathcal{C}$ collects Coriolis–centrifugal terms, $g(q)$ is the gravity vector, and (w, τ) are the base wrench and joint torques. While well-established, this form obscures two important structures: (i) the *energetic structure*—which terms conserve energy versus inject or dissipate it—is hidden in $\mathcal{C}$, requiring auxiliary properties (e.g., skew-symmetry of $\dot{\mathcal{M}} - 2\mathcal{C}$) for passivity analysis; and (ii) the *geometric origin* of terms in $\mathcal{C}$—from the base Lie–Poisson structure, the manipulator symplectic structure, or their coupling—is not transparent.

The port-Hamiltonian framework and its potential. The port-Hamiltonian framework [2] extends Hamiltonian mechanics to open systems exchanging energy through power ports. A port-Hamiltonian system has the form

$$\dot{x} = \mathcal{J}(x)\frac{\partial \mathcal{H}}{\partial x} + \mathcal{G}u, \qquad y = \mathcal{G}^\top \frac{\partial \mathcal{H}}{\partial x}, \qquad (2)$$

where x is the state, $\mathcal{H}$ is the Hamiltonian (total stored energy), $\mathcal{J} = -\mathcal{J}^\top$ is the skew-symmetric interconnection matrix, $\mathcal{G}$ is the input matrix, and the pair (u, y) defines a power port satisfying $\dot{\mathcal{H}} = y^\top u$. This explicit energy structure provides concrete advantages:

- **Control:** Passivity and the power balance enable energy-based controller design: impedance control [3], energy-shaping [4], control by interconnection [5].
- **Simulation:** The Hamiltonian structure admits energy-preserving numerical integrators [6].
- **Learning:** The port-Hamiltonian structure provides a physics-informed inductive bias for learning robot dynamics, e.g., the Port-Hamiltonian Neural ODE networks of [7] on Lie groups guarantee energy conservation by construction.

Despite these merits, a systematic first-principles port-Hamiltonian formulation of VMS has been absent. This work fills that gap; the full derivations with complete proofs are available in [8].

2. Principal Bundle Structure of VMS

The configuration space of a VMS is $Q = G_b \times Q_m$, where G_b is the b-dimensional Lie group of the base pose (e.g., $SE(3)$ with $b = 6$) and $Q_m \cong \mathbb{R}^n$ is the n-dimensional joint space of the manipulator. The kinetic energy Lagrangian is

$$\mathcal{L}_{\text{kin}}(\mathrm{v},q,\dot{q}) = \frac{1}{2}\begin{pmatrix}\mathrm{v}\\ \dot{q}\end{pmatrix}^\top \mathcal{M}(q)\begin{pmatrix}\mathrm{v}\\ \dot{q}\end{pmatrix}, \; \mathcal{M}(q) = \begin{pmatrix}M_b & M_{bm}\\ M_{bm}^\top & M_m\end{pmatrix}, \qquad (3)$$

where $\mathrm{v} \in \mathfrak{g}_b$ is the base body velocity in the Lie algebra $\mathfrak{g}_b$ of G_b, $\dot{q} \in T_qQ_m$ collects the joint velocities, $M_b(q)$ and $M_m(q)$ are the locked and manipulator inertia matrices, and $M_{bm}(q)$ is the coupling inertia.

The configuration space Q of a VMS has the special strcture of a *trivial principal bundle* with base Q_m and structure group G_b (Fig. 1). This decomposes VMS motion into *internal* motions $\dot{q}$ and *external* motions described

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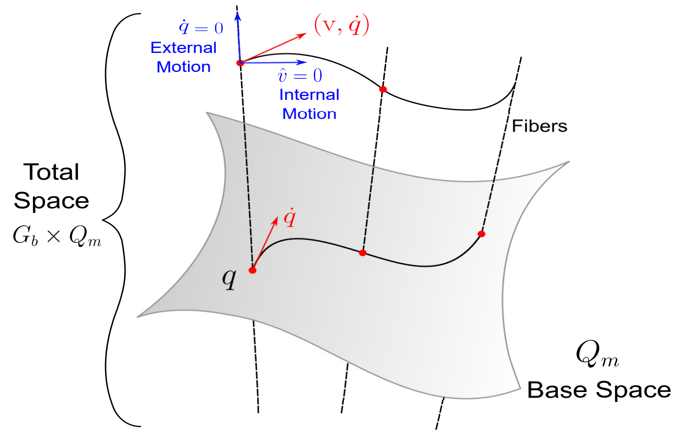

Figure 1: Principal bundle structure of the VMS configuration space.

by the *locked velocity* $\hat{\mathrm{v}} := \mathrm{v} + A(q)\dot{q} \in \mathfrak{g}_b$, where $A(q) := M_b^{-1}(q)\, M_{bm}(q)$ is the *natural local mechanical connection* [9]. In these variables, the Lagrangian becomes inertially decoupled:

$$\hat{\mathcal{L}}_{\text{kin}}(\hat{\mathrm{v}}, q, \dot{q}) = \frac{1}{2}\hat{\mathrm{v}}^\top M_b(q)\, \hat{\mathrm{v}} + \frac{1}{2}\dot{q}^\top \hat{M}_m(q)\, \dot{q}, \tag{4}$$

where $\hat{M}_m(q) := M_m - M_{bm}^\top M_b^{-1} M_{bm}$ is the Schur complement of M_b in $\mathcal{M}$.

3. Inertially-Decoupled port-Hamiltonian Model

Via Hamiltonian reduction on the cotangent bundle T^*Q, we derive the standard port-Hamiltonian dynamics with state $x := (\mathrm{p}, q, \pi)$, where $\mathrm{p} \in \mathfrak{g}_b^*$ is the base momentum in the dual of the Lie algebra and $\pi \in T_q^* Q_m$ is the conjugate momentum of the manipulator:

$$\begin{pmatrix} \dot{\mathrm{p}} \\ \dot{q} \\ \dot{\pi} \end{pmatrix} = \mathcal{J}(\mathrm{p}) \begin{pmatrix} \frac{\partial \mathcal{H}_{\text{kin}}}{\partial \mathrm{p}} \\ \frac{\partial \mathcal{H}_{\text{kin}}}{\partial q} \\ \frac{\partial \mathcal{H}_{\text{kin}}}{\partial \pi} \end{pmatrix} + \mathcal{G} \begin{pmatrix} \mathrm{w} \\ \tau \end{pmatrix}, \tag{5}$$

where $\mathcal{H}_{\text{kin}} = \frac{1}{2}(\mathrm{p}, \pi)^\top \mathcal{M}^{-1}(q)(\mathrm{p}, \pi)^\top$, $\mathcal{G}$ is the input matrix, and $\mathcal{J}(\mathrm{p}) = \begin{pmatrix} ad_{\mathrm{p}}^{\sim} & \cdot & \cdot \\ \cdot & \cdot & \mathbb{I}_n \\ \cdot & -\mathbb{I}_n & \cdot \end{pmatrix}$ is a block-diagonal composition of the base Lie–Poisson structure (encoded by the skew-symmetric coadjoint operator $ad_{\mathrm{p}}^{\sim}$) and the manipulator's canonical symplectic structure ($\mathbb{I}_n$ blocks); the only coupling enters through $\mathcal{M}(q)$ in the Hamiltonian. While (5) serves passivity analysis, the coupling M_{bm} makes the Hamiltonian gradients cumbersome. Exploiting the principal bundle via the coordinate transformation $\hat{q} := q$, $\hat{\mathrm{p}} := \mathrm{p}$, $\hat{\pi} := \pi - A^\top(q)\,\mathrm{p}$, we obtain the *inertially-decoupled port-Hamiltonian dynamics*:

$$\begin{pmatrix} \dot{\hat{\mathrm{p}}} \\ \dot{\hat{q}} \\ \dot{\hat{\pi}} \end{pmatrix} = \hat{\mathcal{J}}(\hat{\mathrm{p}}, \hat{q}) \begin{pmatrix} \frac{\partial \hat{\mathcal{H}}_{\text{kin}}}{\partial \hat{\mathrm{p}}} \\ \frac{\partial \hat{\mathcal{H}}_{\text{kin}}}{\partial \hat{q}} \\ \frac{\partial \hat{\mathcal{H}}_{\text{kin}}}{\partial \hat{\pi}} \end{pmatrix} + \hat{\mathcal{G}}(\hat{q}) \begin{pmatrix} \mathrm{w} \\ \tau \end{pmatrix}, \tag{6}$$

with the *decoupled* Hamiltonian

$$\hat{\mathcal{H}}_{\text{kin}} = \frac{1}{2}\hat{\mathrm{p}}^\top M_b^{-1}(\hat{q})\, \hat{\mathrm{p}} + \frac{1}{2}\hat{\pi}^\top \hat{M}_m^{-1}(\hat{q})\, \hat{\pi}, \tag{7}$$

and the interconnection and input matrices

$$\hat{\mathcal{J}} = \begin{pmatrix} ad_{\hat{\mathrm{p}}}^{\sim} & \cdot & -ad_{\hat{\mathrm{p}}}^{\sim} A \\ \cdot & \cdot & \mathbb{I}_n \\ -A^\top ad_{\hat{\mathrm{p}}}^{\sim} & -\mathbb{I}_n & -\boldsymbol{B} \end{pmatrix}, \ \hat{\mathcal{G}} = \begin{pmatrix} \mathbb{I}_b & \cdot \\ \cdot & \cdot \\ -A^\top & \mathbb{I}_n \end{pmatrix}, \tag{8}$$

where $\boldsymbol{B}(\hat{\mathrm{p}}, \hat{q})$ is a skew-symmetric matrix encoding the curvature of the mechanical connection.

The key advantages of the port-Hamiltonian formulation (6) are: (i) the Hamiltonian (7) separates into base and manipulator contributions, avoiding inversion of the full $\mathcal{M}(q)$; (ii) each entry of $\hat{\mathcal{J}}$ has a clear geometric origin—diagonal blocks from the Lie–Poisson and symplectic structures, off-diagonal blocks from $A(q)$; and (iii) the port-Hamiltonian form directly yields the power balance $\dot{\hat{\mathcal{H}}}_{\text{kin}} = \langle \mathrm{w} | \mathrm{v} \rangle + \langle \tau | \dot{q} \rangle$, where $\langle \cdot | \cdot \rangle$ denotes the dual pairing, confirming passivity w.r.t. (w, v) and $(\tau, \dot{q})$.

Equivalence to Lagrangian Counterparts in the Literature We established rigorous equivalence with the reduced Euler–Lagrange equations of [10, 11] and the Boltzmann–Hamel equations of [12]. A key insight is that each Coriolis–centrifugal term in the Lagrangian formulation can be traced to a specific geometric origin in the port-Hamiltonian model: the symplectic structure of the manipulator, the Lie–Poisson structure of the base, or the tangent and cotangent maps of the bundle coordinates. In the Lagrangian setting these terms are mixed together in the Coriolis matrices, obscuring their origins. Furthermore, passivity analysis in the port-Hamiltonian framework follows directly from the skew-symmetry of $\hat{\mathcal{J}}$, whereas the Lagrangian route requires verifying auxiliary relations between $\mathcal{M}$ and C; this transparency extends to advanced control synthesis such as adaptive and observer-based designs and can be extended to structure-preserving numerical integration and physics-informed learning.

4. Conclusion

We presented a first-principles port-Hamiltonian formulation of VMS exploiting the principal bundle structure via Hamiltonian reduction. The inertially-decoupled model makes the energy structure, power flow, and geometric origin of every dynamic term explicit. This opens avenues for energy-based control [13, 5], exploiting the symmetry of VMS for control [14], structure-preserving simulation [6], and geometric-based learning methods [15, 7]. The coordinate-free approach avoids singularities of local parameterizations, and the Dirac structure formalism can be extended naturally to include holonomic and nonholonomic constraints, thus providing a unified framework for a wider class of robotic systems.

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